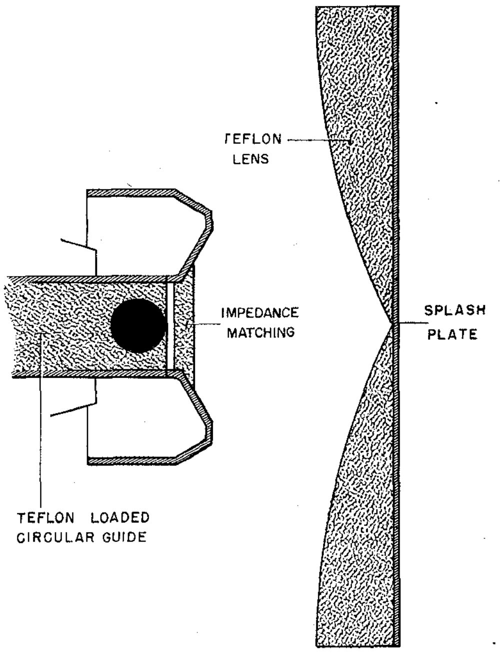

Fig. 4—Ferrite application to a feed for a conical scanning antenna.

only 10 db below the normal polarization. Further side lobes are up to 15 db below the peak of the beam, and antenna efficiency is low. Nevertheless, the device demonstrates an application which could produce a conical scanner in principle in all respects including the problem of reciprocity in the ferrite.

Conclusion

It has been shown that a small sphere immersed in an oblique uniform magnetic field and an rf circularly polarized plane wave will scatter electromagnetic energy principally along the magnetic field. It is demonstrated that this principle can be used to direct a wide radiated beam at the expense of some depolarization of the incident wave. Finally, it has been shown as an example that this could be applied to the problem of conical scanning although a scanner has not been made which could compete with present mechanical scanners.

Acknowledgment

Most of the measurements described were made by R. Cox of the Westinghouse Air Arm Division.

A Ferrite Boundary-Value Problem in a Rectangular Waveguide*

C. B. SHARPE† AND D. S. HEIM†

Summary—**A solution is obtained for the electric field at the air-ferrite interface ($z=0$) in a rectangular waveguide filled with ferrite in the semi-infinite half ($z>0$) and magnetized in the direction of the electric field. The field is expressed in terms of a Neumann series obtained by iteration of a singular integral equation which satisfies the boundary conditions at the interface. The equivalent circuit for the junction is also presented.**

Introduction

THE mathematical difficulties which are encountered in the solution of many boundary value problems involving gyromagnetic media have been pointed out by several authors.[1-3] The formulation of these problems is usually straightforward but the imposition of the boundary conditions at the isotropic-to-anisotropic interface frequently makes them intractable. The one discussed here, in which the anisotropic media is a semi-infinite slab of ferrite filling a rectangular waveguide, appears to present some of the essential difficulties common to the solution of many such problems.

Referring to Fig. 1, we consider an infinite rectangular waveguide which is filled with a ferrite medium for $z>0$ and air for $z<0$. The ferrite region is magnetized in the y direction with an internal field H. A TE_{10} wave is incident from the left at the air-ferrite interface ($z=0$). The problem is to determine the electric and magnetic fields at the interface and the equivalent circuit for the junction. The ferrite medium is assumed to be lossless and characterized by a tensor permeability

$$(\mu) = \begin{pmatrix} \mu & 0 & j\kappa \\ 0 & \mu_0 & 0 \\ -j\kappa & 0 & \mu \end{pmatrix}$$

where

* Manuscript received by PGMTT, May 24, 1957; revised manuscript received, August 12, 1957. This work was sponsored by the Signal Corps Eng. Labs., Dept. of the Army, Contract No. DA-36-039 sc-63203.

† Electronic Defense Group, Dept. of Elec. Eng., Univ. of Michigan, Ann Arbor, Mich.

[1] P. S. Epstein, "Theory of wave propagation in a gyromagnetic medium," *Rev. Mod. Phys.*, vol. 28, pp. 3–17; January, 1956.

[2] A. A. Th. Van Trier, "Guided electromagnetic waves in anisotropic media," *Appl. Sci. Res.*, vol. 3, sec. B, pp. 305–370; 1953.

[3] H. Suhl and L. R. Walker, "Topics in guided wave propagation through gyromagnetic media," *Bell Sys. Tech. J.*, vol. 33, Part I, pp. 579–659; May, 1954, Part II, pp. 939–986; July, 1954, and Part III, pp. 1133–1194; September, 1954.

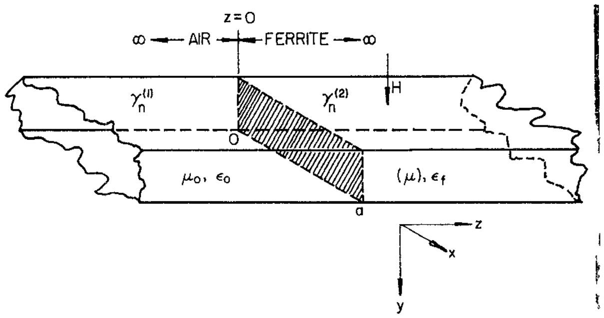

Fig. 1—Ferrite-filled waveguide.

$$\frac{\mu}{\mu_0} = 1 + \frac{\Gamma^2 M_s H}{\Gamma^2 H^2 - \omega^2},$$

$$\frac{\kappa}{\mu_0} = \frac{\omega \Gamma M_s}{\Gamma^2 H^2 - \omega^2}.$$

In rationalized mks units, which are used throughout, the gyromagnetic ratio is given by

$$\Gamma = -0.22 \times 10^6 \text{ m/ampere-second.}$$

M_s is defined as the magnetization at saturation using the convention

$$B = \mu_0(H + M).$$

All field quantities are taken proportional to exp $(j\omega t)$.

The Equivalent Circuit

Assuming all field quantities are independent of the y coordinate, the transverse electric and magnetic fields in the ferrite [medium (2)] can be expressed by[4]

$$E_y^{(t)} = \sum_{n=1}^{\infty} T_n \sin n\pi x/a \, e^{-\gamma_n^{(2)} z}, \quad (1)$$

$$H_x^{(t)} = \sum_{n=1}^{\infty} T_n[nM \cos n\pi x/a - Y_n^{(2)} \sin n\pi x/a] e^{-\gamma_n^{(2)} z}, \quad (2)$$

where

$$\gamma_n^{(2)} = \sqrt{(n\pi/a)^2 - \omega^2 \mu_\perp \epsilon_f}, \qquad \mu_\perp = \frac{\mu^2 - \kappa^2}{\mu},$$

$$M = \frac{\pi}{a\omega} \frac{\kappa}{\mu^2 - \kappa^2}, \qquad Y_n^{(2)} = \frac{\gamma_n^{(2)}}{j\omega\mu_\perp}.$$

In the air-filled section [medium (1)] the incident TE_{10} wave is given by

$$E_y^{(i)} = \sin \pi x/a \, e^{-\gamma_1^{(1)} z} \quad (3)$$

$$H_x^{(i)} = -Y_1^{(1)} \sin \pi x/a \, e^{-\gamma_1^{(1)} z}, \quad (4)$$

and the reflected TE_{10} waves are given by

$$E_y^{(r)} = \sum_{n=1}^{\infty} R_n \sin n\pi x/a \, e^{\gamma_n^{(1)} z} \quad (5)$$

$$H_x^{(r)} = \sum_{n=1}^{\infty} Y_n^{(1)} R_n \sin n\pi x/a \, e^{\gamma_n^{(1)} z}, \quad (6)$$

[4] *Ibid.*, Part II.

where

$$Y_n^{(1)} = \frac{\gamma_n^{(1)}}{j\omega\mu_0}, \qquad \gamma_n^{(1)} = \sqrt{(n\pi/a)^2 - \omega^2 \mu_0 \epsilon_0}.$$

The necessity of (5) and (6) is discussed by Epstein.[5] Satisfying the boundary conditions at the interface $z=0$ and eliminating the R_n results in the following for the coefficients of the electric field:

$$[T_1(Y_1^{(1)} + Y_1^{(2)}) - 2Y_1^{(1)}] \sin \pi x/a$$

$$= \sum_{n=1}^{\infty} nMT_n \cos n\pi x/a$$

$$- \sum_{n=2}^{\infty} (Y_n^{(1)} + Y_n^{(2)}) T_n \sin n\pi x/a, \quad 0 \le x \le a. \quad (7)$$

Both Epstein[5] and Van Trier[6] have pointed out that (7) leads to an infinite system of simultaneous linear equations for which there is no practicable method of solution. Epstein has used a method of successive approximations to obtain a power-series expansion for R_1 in terms of $\kappa/(\mu^2-\kappa^2)$. However, as he points out, this leaves much to be desired, particularly since $\kappa/(\mu^2-\kappa^2)$ can be very large. In the solution which follows, (7) will be expressed as an integral equation in terms of the electric field E_y at the interface. The solution will yield the electric and magnetic fields directly for almost all values of $\kappa/(\mu^2-\kappa^2)$.

It will be convenient to normalize (7). Consider first the case where only the dominant mode propagates in the ferrite medium. That is,

$$(\pi/a)^2 < \omega^2 \mu_\perp \epsilon_f < (2\pi/a)^2.$$

It is assumed in all cases that only the dominant mode propagates in the air-filled section. The transmission line circuit illustrated in Fig. 2 will be equivalent to the waveguide junction if we can identify the voltage and current waves on the line with the fundamental components of the transverse electric and magnetic fields, respectively, in the waveguide. The analogy which we shall employ here makes the constant of proportionality between current and voltage equal to the wave admittance for the dominant mode in the corresponding waveguide. The quantity Re $[V^{(j)} I^{(j)*}]$, $j=i, r, t$, (Fig. 2) will be proportional to the power flow in the corresponding waveguide. Thus, in the air-filled section,

$$E_{y1}^{(i)} = V(z)^{(i)} \sin \pi x/a, \quad E_{y1}^{(r)} = V(z)^{(r)} \sin \pi x/a$$

$$H_{x1}^{(i)} = -Y_1^{(1)} E_{y1}^{(i)} = I(z)^{(i)} \sin \pi x/a,$$

$$H_{x1}^{(r)} = Y_1^{(1)} E_{y1}^{(r)} = I(z)^{(r)} \sin \pi x/a$$

where the subscript 1 denotes the first mode. In the ferrite-filled section a fundamental difficulty occurs since $H_{x1}^{(t)}$ is not simply proportional to $E_{y1}^{(t)}$. Nevertheless, a similar correspondence can be made:

[5] Epstein, *op. cit.*, p. 14.
[6] Van Trier, *op. cit.*, p. 335.

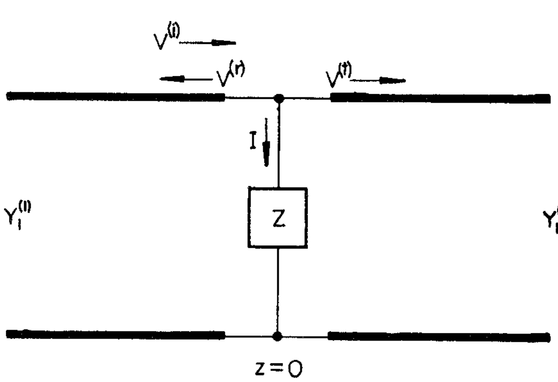

Fig. 2—Equivalent circuit for the air-ferrite junction.

$$E_{y1}{}^{(t)} = V(z)^{(t)} \sin \pi x/a$$
$$Y_1{}^{(2)}E_{y1}{}^{(t)} = I(z)^{(t)} \sin \pi x/a.$$

Power flow in the ferrite medium is still proportional to Re $[V^{(t)}I^{(t)*}]$ since the cosine term in (2) does not contribute to the integral of Poynting's vector over the cross section of the waveguide. Of course, the continuity of H_x at the junction no longer implies the continuity of current flow in the equivalent circuit. This discontinuity in current flow at the junction is accounted for by the current I flowing through the impedance Z.

Since, at $z=0$

$$V^{(i)} = 1, \qquad V^{(r)} = R_1, \qquad V^{(t)} = T_1,$$

it follows from Fig. 2 that

$$-I = T_1(Y_1{}^{(1)} + Y_1{}^{(2)}) - 2Y_1{}^{(1)}. \tag{8}$$

It will be useful to make the change of variable, $\phi = \pi x/a$. Then, at $z=0$

$$E_y{}^{(t)}(\phi) = \sum_{n=1}^{\infty} T_n \sin n\phi, \qquad 0 \le \phi \le \pi, \tag{9}$$

where

$$T_n = \frac{2}{\pi}\int_0^{\pi} E_y{}^{(t)}(\phi) \sin n\phi d\phi. \tag{10}$$

Following Miles,[7] we define a normalized field proportional to $E_y{}^{(t)}$,

$$E_y{}^{(t)}(\phi) = I\mathcal{E}(\phi). \tag{11}$$

Whence,

$$z = JX = \frac{V^{(t)}}{I} = \frac{2}{\pi}\int_0^{\pi} \mathcal{E}(\phi) \sin \phi d\phi.$$

It can be shown[8] that for a ferrite described by a Hermitian tensor the Re $[Z]=0$, which confirms the assumption of a lossless medium. Eq. (7) can now be written,

$$-\sin\phi = M\mathcal{E}'(\phi) - \frac{2}{\pi}\sum_{n=2}^{\infty}(Y_n{}^{(1)} + Y_n{}^{(2)})$$
$$\cdot \int_0^{\pi} \mathcal{E}(\phi') \sin n\phi' \sin n\phi d\phi', \quad 0 \le \phi \le \pi. \tag{13}$$

$\mathcal{E}'(\phi)$ denotes the derivative of $\mathcal{E}(\phi)$ with respect to ϕ.

When all modes are cut off in the ferrite,

$$\omega^2\mu_\perp\epsilon_f < (\pi/a)^2,$$

and $Y_1{}^{(2)}$ is imaginary. The equivalent circuit of Fig. 2 will suffice for this case also with the understanding that no power is transmitted away from the junction to the right. We cannot neglect $I^{(t)}$ since it gives rise to an inductive susceptance in parallel with Z. The impedance Z accounts for the discontinuity in the sinusoidal component of H_x as before. The theory which follows will therefore be valid for both the case where only the dominant mode propagates in the ferrite and the case where all modes are cut off.

Theory

In order to solve (13) for $\mathcal{E}(\phi)$, it is necessary to make a commonly used assumption; namely,

$$\gamma_n{}^{(1)} = \gamma_n{}^{(2)} \cong n\pi/a, \qquad n > 1.$$

Then $Y_n{}^{(1)} + Y_n{}^{(2)}$ can be approximated by

$$Y_n{}^{(1)} + Y_n{}^{(2)} \cong -jKn, \qquad n > 1, \tag{14}$$

where

$$K = \frac{\pi}{a\omega}[1/\mu_0 + 1/\mu_\perp].$$

Eq. (14) is usually a reasonable assumption to make for problems where only the first mode propagates. However, we shall find in the present problem that this assumption appears to be of critical importance for the case where $M/K=1$.

With this assumption, (13) can be written,

$$C\sin\phi$$
$$= M\mathcal{E}'(\phi) + jK\frac{2}{\pi}\sum_{n=1}^{\infty} n\int_0^{\pi}\mathcal{E}(\phi')\sin n\phi' \sin n\phi d\phi', \tag{15}$$

where

$$-C = 1 + jK\frac{2}{\pi}\int_0^{\pi}\mathcal{E}(\phi)\sin\phi d\phi = 1 - KX. \tag{16}$$

Integrating the last term in (15) by parts and employing the identity[9]

$$\sin n\phi = \frac{1}{\pi}\int_0^{\pi}\frac{\sin\phi\cos n\psi}{\cos\psi - \cos\phi}d\psi, \quad (n = 0, 1, 2, \cdots),$$

[7] J. W. Miles, "The equivalent circuit for a plane discontinuity in a cylindrical wave guide," Proc. IRE, vol. 34, pp. 728–742; October, 1946.

[8] C. B. Sharpe and D. S. Heim, "Reflections in a Ferrite Filled Waveguide," Univ. of Michigan, Electronic Defense Group, Tech. Rep. No. 72; May, 1957.

[9] W. Magnus and F. Oberhettinger, "Formulas and Theorems for the Functions of Mathematical Physics," Chelsea Publishing Co., New York, N. Y., p. 141; 1954.

one obtains

$$C \sin \phi = M\mathcal{E}'(\phi) + jK \frac{2}{\pi^2} \sin \phi \sum_{n=1}^{\infty} \int_0^{\pi} \mathcal{E}'(\phi') \cos n\phi' d\phi' \cdot \int_0^{\pi} \frac{\cos n\psi}{\cos \psi - \cos \phi} d\psi. \quad (17)$$

Interchanging the order of integration and summation in (17) results in a singular integral equation of the second kind.

$$C \sin \phi = M\mathcal{E}'(\phi) + j \frac{K}{\pi} \int_0^{\pi} \frac{\mathcal{E}'(\psi) \sin \phi}{\cos \psi - \cos \phi} d\psi. \quad (18)$$

It is convenient to define

$$\mathcal{E}(\phi) = \mathcal{E}_r(\phi) + j\mathcal{E}_i(\phi).$$

Eq. (18) can then be expressed by the system,

$$C \sin \phi = M\mathcal{E}_r'(\phi) - \frac{K}{\pi} \int_0^{\pi} \frac{\mathcal{E}_i'(\psi) \sin \phi}{\cos \psi - \cos \phi} d\psi \quad (19)$$

$$0 = M\mathcal{E}_i'(\phi) + \frac{K}{\pi} \int_0^{\pi} \frac{\mathcal{E}_r'(\psi) \sin \phi}{\cos \psi - \cos \phi} d\psi. \quad (20)$$

Solution for Small M/K

Schmeidler[10] has shown how the system of integral (19) and (20) can be solved by a process of iteration when $|M/K| < 1$. We shall have need for the following integral equation and its solution:

$$f(s) = \frac{1}{\pi} \int_0^{\pi} \frac{\sin s}{\cos t - \cos s} g(t) dt \quad (21)$$

$$g(t) = \frac{1}{\pi} \int_0^{\pi} g(t) dt - \frac{1}{\pi} \int_0^{\pi} \frac{f(s) \sin s}{\cos s - \cos t} ds. \quad (22)$$

Recalling that $\mathcal{E}(\phi) = 0$ for $\phi = 0, \pi$, the application of (21) and (22) to (19) and (20) yields

$$\mathcal{E}_i'(\phi) = -\frac{C}{K} \cos \phi - (M/K)^2 \frac{1}{\pi^2} \int_0^{\pi} \frac{\sin \tau d\tau}{\cos \phi - \cos \tau} \cdot \int_0^{\pi} \frac{\mathcal{E}_i'(\psi) \sin \psi}{\cos \tau - \cos \psi} d\psi. \quad (23)$$

Eq. (23) can be identified as an integral equation of the form

$$\mathcal{E}_i'(\phi) = f(\phi) + \lambda \int_0^{\pi} K(\phi, \tau) d\tau \int_0^{\pi} \mathcal{E}_i'(\psi) K(\tau, \psi) d\psi \quad (24)$$

by making the correspondence,

$$K(\phi, \tau) = \frac{1}{\pi} \frac{\sin \tau}{\cos \phi - \cos \tau}$$

$$f(\phi) = -\frac{C}{K} \cos \phi$$

$$\lambda = -(M/K)^2. \quad (25)$$

[10] W. Schmeidler, "Integralgleichungen mit Anwendungen in Physik und Technik," Geest and Portig K.-G., Leipzig, Ger., 1955.

Iterating (24) yields,

$$\mathcal{E}_i'(\phi) = f(\phi) + \lambda \int_0^{\pi} K(\phi, \tau) d\tau \int_0^{\pi} f(\psi) K(\tau, \psi) d\psi + \lambda^2 \int_0^{\pi} K(\phi, \tau) d\tau \int_0^{\pi} K(\tau, \sigma) d\sigma \int_0^{\pi} K(\sigma, \rho) d\rho \, \mathcal{E}_i'(\psi) K(\rho\psi) d\psi.$$

Repeating the process leads to a Neumann series for $\mathcal{E}_i'(\phi)$. It can be shown[11] that for the kernel (25), the above series converges for $|\lambda| < 1$. Thus, the first order approximation to $\mathcal{E}'(\phi)$ can be taken as

$$\mathcal{E}_{i_1}'(\phi) = f(\phi)$$

and the second order approximation as

$$\mathcal{E}_{i_2}'(\phi) = f(\phi) + \lambda \int_0^{\pi} K(\phi, \tau) d\tau \int_0^{\pi} f(\psi) K(\tau, \psi) d\psi. \quad (26)$$

We shall obtain only the second order approximation, although the extension to higher order approximations is obvious. It follows that

$$\mathcal{E}_{i_2}'(\phi) = -\frac{C}{K}\left[1 + \frac{1}{2}\left(\frac{M}{K}\right)^2\right] \cos \phi - \frac{2C}{\pi^2 K}\left(\frac{M}{K}\right)^2 \ln \frac{1 + \cos \phi}{1 - \cos \phi} + \frac{C}{2\pi^2 K}\left(\frac{M}{K}\right)^2 \cos \phi \left(\ln \frac{1 + \cos \phi}{1 - \cos \phi}\right)^2, \quad \left|\frac{M}{K}\right| < 1. \quad (27)$$

The solution for $\mathcal{E}_r'(\phi)$ follows in the same manner. There results

$$\mathcal{E}_{r_2}'(\phi) = -\frac{C}{\pi K}\left[\frac{5}{3}\left(\frac{M}{K}\right)^3 + 2\left(\frac{M}{K}\right)\right] + \frac{C}{\pi K}\left[\left(\frac{M}{K}\right) + \frac{5}{6}\left(\frac{M}{K}\right)^3\right] \cos \phi \left(\ln \frac{1 + \cos \phi}{1 - \cos \phi}\right) + \frac{C}{\pi^3 K}\left(\frac{M}{K}\right)^3 \left[\left(\ln \frac{1 + \cos \phi}{1 - \cos \phi}\right)^2 - \frac{1}{6} \cos \phi \left(\ln \frac{1 + \cos \phi}{1 - \cos \phi}\right)^3\right], \quad \left|\frac{M}{K}\right| < 1. \quad (28)$$

It remains to determine C, which is a function of the unknown reactance X. From (12),

$$X = \frac{2}{\pi} \int_0^{\pi} \mathcal{E}_i'(\phi) \cos \phi d\phi. \quad (29)$$

[11] *Ibid.*, p. 429.

It follows that

$$C = \frac{\pi^2}{4}\left(\frac{K}{M}\right)^2,$$

$$X = -\frac{1}{K}\left\{\frac{1+\dfrac{4}{\pi^2}\left(\dfrac{M}{K}\right)^2}{\dfrac{4}{\pi^2}\left(\dfrac{M}{K}\right)^2}\right\}, \qquad \left|\frac{M}{K}\right| < 1. \tag{30}$$

Solution for Large M/K

A Neumann series valid for $|M/K| > 1$ can also be obtained from (19) and (20). By direct substitution,

$$\mathcal{E}_i'(\phi) = \frac{C}{\pi M}\left(\frac{K}{M}\right)\sin\phi\left(\ln\frac{1+\cos\phi}{1-\cos\phi}\right)$$
$$-\left(\frac{K}{M}\right)^2\frac{1}{\pi^2}\int_0^\pi\frac{\sin\phi d\tau}{\cos\phi-\cos\tau}\int_0^\pi\frac{\mathcal{E}_i'(\psi)\sin\tau}{\cos\tau-\cos\psi}d\psi \tag{31}$$

$$\mathcal{E}_r'(\phi) = \frac{C}{M}\sin\phi$$
$$-\left(\frac{K}{M}\right)^2\frac{1}{\pi^2}\int_0^\pi\frac{\sin\phi d\tau}{\cos\phi-\cos\tau}\int_0^\pi\frac{\mathcal{E}_r'(\psi)\sin\tau}{\cos\tau-\cos\psi}d\psi. \tag{32}$$

Both (31) and (32) reduce to an equation of the same form as (24) if we take

$$K(\phi,\tau) = \frac{1}{\pi}\frac{\sin\phi}{\cos\phi-\cos\tau}$$

$$\lambda = -\left(\frac{K}{M}\right)^2.$$

Iteration again yields a Neumann series in terms of λ. It can be shown that the series will converge for $|\lambda| < 1$, that is, for $1 < |M/K|$. The proof parallels that given by Schmeidler for the previous case. Second order approximations to $\mathcal{E}_i'(\phi)$ and $\mathcal{E}_r'(\phi)$ are found to be,

$$\mathcal{E}_{i_2}'(\phi) = \frac{C}{\pi K}\left[\left(\frac{K}{M}\right)^2+\frac{5}{6}\left(\frac{K}{M}\right)^4\right]\sin\phi\left(\ln\frac{1+\cos\phi}{1-\cos\phi}\right)$$
$$-\frac{1}{6\pi^3}\frac{C}{K}\left(\frac{K}{M}\right)^4\sin\phi\left(\ln\frac{1+\cos\phi}{1-\cos\phi}\right)^3 \tag{33}$$

$$\mathcal{E}_{r_2}'(\phi) = \frac{C}{K}\left[\left(\frac{K}{M}\right)+\frac{1}{2}\left(\frac{K}{M}\right)^3\right]\sin\phi$$
$$-\frac{C}{2\pi^2 K}\left(\frac{K}{M}\right)^3\sin\phi\left(\ln\frac{1+\cos\phi}{1-\cos\phi}\right)^2. \tag{34}$$

The constant C and the reactance X for this case are

$$C = \frac{-1}{1+\dfrac{4}{\pi^2}\left[\left(\dfrac{K}{M}\right)^2+\dfrac{2}{3}\left(\dfrac{K}{M}\right)^4\right]},$$

$$X = \frac{-1}{K}\left\{\frac{\left(\dfrac{K}{M}\right)^2+\dfrac{2}{3}\left(\dfrac{K}{M}\right)^4}{\dfrac{\pi^2}{4}+\left(\dfrac{K}{M}\right)^2+\dfrac{2}{3}\left(\dfrac{K}{M}\right)^4}\right\}, \qquad \left|\frac{K}{M}\right| < 1. \tag{35}$$

Discussion

It is interesting to note that the example discussed by Schmeidler which gives rise to (19) and (20) is a problem in elasticity. The real and imaginary components of $\mathcal{E}'(\phi)$ are analogous to the horizontal and vertical components of pressure, respectively, at the base of a dam. The depth of water as a variable has the same significance as the magnitude of the magnetizing field H in the electromagnetic field problem.

The existence of both a real and imaginary part to the field at the interface is unique to boundary-value problems involving anisotropic media. Although the field strength at each position across the waveguide varies sinusoidally with time, the phase of this variation differs from one point to the next. Thus, the field exhibits a periodic "shimmy" in time.

The value of $|M/K| = 1$ seems to be a critical point in the analysis. Not only do the fields display a marked difference in form for values of M/K on either side of unity but the series solution itself probably does not converge for this critical value. One would suspect the assumption of (14) to be the source of the difficulty.

The value of $|M/K| = 0$ leads to an indeterminate solution for the normalized field $\mathcal{E}_i'(\phi)$ because the normalizing factor I is also zero at this point. This is of little consequence, however, since for $M = 0$ the problem reduces to the case of an isotropic dielectric. It is interesting to note, again in contrast to isotropic problems that X may be either inductive or capacitive since K can be negative or positive.